INTRODUCTION

Presumably the most significant drawback of the conventional FDTD method in electromagnetics is the linear relation between the discretization steps in space and time. If an electrically small structure makes it necessary to refine the FDTD grid by a factor r in all three spatial dimensions, the total increase in computational resources ranges between r^2 and r^4 . Thus, the method as is can not be used to model structures with relevant dimensions much smaller than the wavelength.

One of the subgroups of electrically small objects is the set of thin conductive (TC) sheets. Multiple approaches to treat TC and thin layers in general with the FDTD method have been reported, an overview can be found in [1] and [2]. However, they all suffer from showing results for flat, grid-aligned sheet structures only. Moreover, the common methodology underlying all of those approaches is that they start from physics equations, namely Maxwell's equations, to find a discretized form that fits more or less successfully into the FDTD scheme. By introducing additional degrees of freedom (DOF), i.e., additional grid components or state variables, the proven robustness and implementational feasibility are no longer guaranteed. Therefore, problems such as instabilities or unmanageable implementational burdens leave those approaches unfit for professional and commercial use in 3D EM FDTD applications.

METHODOLOGY

The presented algorithm is following a new approach. We propose that, before resorting to the enhancement of the regular FDTD update equations, one should ask if the number of DOF intrinsically available in the Yee scheme is already sufficient to model a specific phenomenon.

In the context of TC sheets, we start with the analytical description of a plane wave incident on a TC sheet and ask if we can match its discretized representation with the standard FDTD equations by choosing appropriate update coefficients.

Combining the analytical solution of a plane wave normally incident on a flat sheet and the common FDTD update equations leads to the desired coefficients in 1D. To make the step to sheets that may be curved in all dimensions, those update equations are applied to a specific subset of FDTD edges that belong to such a sheet, where each of these edges models a certain polarization.

The result is an efficient way of treating 3D TC sheets within FDTD. The algorithm has been implemented into the fully featured EM FDTD tool SEMCAD X and applied to various canonical and real world problems. Due to the fact that the algorithm works with the unaltered update equations, it could be succesfully integrated into almost all solvers offered by SEMCAD X such as ADI-FDTD and hardware accelerated FDTD.

HOMOGENEOUS, GRID-ALIGNED 1D FORMULATION

Combining the analytical description of a plane wave

$$
E_{in}(t, z) = E_0 e^{i(\omega t - kz)} - Re^{i(\omega t + kz)}
$$

$$
E_{out}(t, z) = TE_0 e^{i(\omega t - kz)}
$$

with the FDTD update equation for a sheet edge

$$
E\mid_0^{n+1/2} = \alpha_S E\mid_0^{n-1/2} + \beta_S \left(H\mid_{-1/2}^n - H\mid_{+1/2}^n \right)
$$

yields the modified update coefficients

This model is valid for flat, grid-aligned sheets in homogeneous grids only. However, numerical experiments with such sheets show that both the influence of the angle of incidence and the surrounding material are treated correctly, an example being shown in Figure 1.

GRADED, CURVED 3D FORMULATION

To extend the method for arbitrarily curved sheets in graded meshes, an algorithm has been developed that analyses the set of E-Field edges by which a 2-dimensional sheet would normally

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Figure 3: Geometric projection for the general case. The grey rectangle represents part of a tilted sheet with local surface normal n. Because the FDTD edge Ex in the staircased FDTD grid (dotted lines) is supposed to model the tangential surface component E_s , the projection modifier C_x is applied.

$$
\alpha_S = c,
$$

$$
\beta_S = c\beta_0
$$

$$
\beta_0 = dT/\epsilon_0 dG.
$$

where the modifier c is given by

 $c = \frac{ZT}{ZT + 2R\beta_0}$

with time step dT , grid step dG , free space impedance Z and reflection and transmission coefficient R and T respectively.

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> be represented in the FDTD grid. This set is reduced such that we are left with a minimal grid, i.e., a grid with the minimal number of edges that could still model a PEC layer by using update coefficients equal to zero. For grid-aligned sheets this transformation is equal to identity. Figure 2 shows an example of a minimal grid taken from the free space bowl simulation (cf. Figure 4).

> > *Figure 7: Flip cell phone having a coated cover at the top. Above: The staircased representation of the thin conductive shee.*

The crucial property of a minimal grid is that each of the remaining E-Field edges is connecting two H-Field edges on either side of the sheet. Thus, each of those edges can then be viewed to model the E-Field component for a specific polarization of the incident field as in the 1D case.

Due to the staircasing the surface of a curved sheet appears to be enlarged. To prevent the error produced by this effect, a geometric projection is applied using the local surface normal of the original 3D object (cf. Figure 3).

The final coefficient modifier c for the general case in 3D writes

$$
c = \frac{C_x Z T}{C_x Z T + 2 R \beta_0' \left[\frac{C_z}{dY} + \frac{C_y}{dZ}\right]} \quad \ \beta_0' = \beta_0 dG = dT/\epsilon
$$

with the projection coefficients C_x and C_y as depicted in Figure 3 and the grid steps dY and dZ . The general formula for c reduces to the 1D case for a flat, grid-aligned sheet. The modifier c fits nicely between the material models PEC ($T=0$) and free space $(R=0)$.

CANONICAL BENCHMARKS

Curved bowl

In order to investigate the performance for curved sheets in nonuniform grids, a canonical benchmark is used. Because most thin sheet structures are electrically too thin to be modeled with standard FDTD for comparison, the range of possible benchmark structures is limited. The geometry used in the presented benchmark consists of a thin conductive sheet shaped like a bowl surrounded by free space as depicted in Figure 4.

This setup proved suitable for our purposes, i.e., a full resolution simulation of the bowl with standard FDTD is possible with commonly available computational resources. Figure 5 shows a comparison of the root mean square (RMS) E-field values measured in dB in the x-y-plane at $y = 0$, i.e., in the very middle of the bowl, for the three different sheet materials. One can see that the three different sheets have a significant impact on the simulation and that the thin conductive sheet simulations are in very good agreement with the fully resolved FDTD comparison simulations despite the differently discretized grids.

Microstrip

To provide an example that does rely on a derived value rather than analyzing and comparing the field values, we used a generic microstrip line of 810 microns in width and 20 microns in thickness over a substrate with a relative permittivity ε =2.7 and a conductivity σ =0.002 S/m. The simple structure allowed to resolve the microstrip with standard FDTD in order to investigate the relation between input impedance and strip parameters. As shown in Figure 6, the results taken from the standard FDTD simulations are in very good agreement with the TCS algorithm.

The simulation time for the fully resolved broadband FDTD simulation was in the order of 4 hours (grid size: 8 million cells), whereas the simulation using the novel TCS algorithm was running for about 1 minute (grid size: 0.15 million cells).

Figure 6: Simulated input impedance of a rectangular microstrip for three different sheet conductivities. The full lines show the results of the fully resolved FDTD simulation, the dotted lines show the results from the simulations using the TCS algorithm.

Figure 2: The minimal grid as produced by the TCS algorithm. The red lines represent staircased edges that have been removed from the original FDTD representation.

*Figure 1: Reflection and transmission coefficients for a plane wave incident on a flat, grid-aligned sheet. The solid and dotted lines show the analytic values, (*x*) and (+) the measurement results. The relevant simulation parameters are: Frequency 1.8GHz; Sheet Thickness 1 micron; Sheet Conductivity 5309 S/m; Relative Dielectric Permittivity* ε=*2 on the outgoing side.*

uctive Sheets

Figure 8: SEMCAD X model of the Nokia 8310. The support frame (red) was coated using a conductive spray, which acts then as a grounding mechanism between main and keypad PCBs.

Figure 9: E-Field Distribution at 902 MHz (V/m) measured 5mm atop the phone casing. With a sheet thickness of 0.5 microns , the transmission factor and conductivity for the four different cases are (from left to right): 75%, 3500 S/m; 50%, 1e4 S/m; 25%, 3e5 S/m; 0%, 1e8 S/m (PEC).

Figure 5: Top to Bottom: The simulated E-Field RMS in dB normalized to 2.5 V/m measured in the x-z-plane at y=0 for three sheet conductivities (1065;265;66 S/m). The left side shows the fully resolved FDTD results using a fine grid of 14 million cells, the right side shows the results obtained with the TCS algorithm using a grid of only 0.8 million cells.

- *[1] Allen Taflove and Susan Hagness, Computational Electrodynamics: The Finite-Difference Time-Domain Method, 2 ed., Artech House, Boston, MA, 2000.*
- *[2] J.G. Maloney and G.S. Smith. A Comparison of methods for modeling electrically thin dielectric and conducting sheets in the finite-difference time-domain method. IEEE Transactions on Antennas and Propagation, vol. 41, no. 5: 690-694, 1993*

APPLICATIONS

Integrated into the simulation platform SEMCAD X the TCS algorithm can be used with arbitrarily complex structures such as cellular phones. An example application is the modeling of phone covers coated with metallic sprays or paints as shown in Figure 7. Such coatings, being extremely thin but with non-zero conductivity, cannot be treated by standard FDTD.

Example Application: Nokia 8310

The multiband (GSM/DCS) phone Nokia 8310 was simulated using SEMCAD X. A discrepancy with the measured results was succesfully alleviated using the novel TCS algorithm. One of the parts, a dielectric support frame shown in Figure 8, was coated with a thin metallic paint to facilitate grounding between the two PC boards. Using the TCS algorithm, a series of simulations with different parameters applied to the coating shows a significant effect: as the properties of the coating become less metallic the grounding between the 2 PCBs is degraded. A significant influencing in the field strength above the antenna can be seen in Figure 9, and hence an influence on the overall radiation performance of the device.

SUMMARY & OUTLOOK

A novel algorithm that extends the applicability of FDTD to include the group of arbitrarily curved, thin metallic sheets has been developed, implemented into a fully featured FDTD EM platform and applied to various benchmarks and real world problems. The core advantage of the approach being that it does not introduce new dependencies to the common FDTD scheme, the TCS update coefficients can be transformed into material parameters. Thus, the algorithm was successfully applied to all FDTD kernels offered by SEMCAD X (FDTD, ADI-FDTD, Hardware Accelerated FDTD) except the conformal variants.

The benchmarks and applications show very good agreement with analytic values and standard FDTD comparison simulations. Because the algorithm does not use all the degrees of freedom offered by the common FDTD scheme, further research may show that the method can be extended to be valid for a broader range of material parameters.

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REFERENCES

Figure 4: Free Space Bowl Benchmark: A bowl of 20 microns thickness surrounded by a plane wave source.